

RF Effect on Beam Emittance

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Simulation Parameters

- MAD Lattice: FNAL-BST Lattice
- Ring Circumference: 474.204810 (m)
- Beam Kinetic Energy: 400 (MeV)
- β_R : 0.7131
- γ_R : 1.4263
- τ_{REV} : 2.2 (μ sec)
- γ_{TR} : 5.4696
- $|\eta|$ at Injection: 0.458
- β_x / β_y : 7.303 / 20.0232 (m)
- α_x / α_y : 0.214 / -0.180 (m)
- Φ_{min}/Φ_{max} : -180.0/180.0 (deg.)
- RF Voltage: 0.0 ~ 205.0 (KV/Turn)
- Transverse Distribution: bi-Gaussian
- Longitudinal Distribution:
 Uniform Distribution
- Max. No. of MacroParticle: 110K
- Injection Turns: 11
- Total Proton Intensity: 6.00e10 (ppb)
- RF Harmonics: 1
- FNAL-BST Harmonic No.: 84
- RF Phase = 0.0 (deg.)
- Tracking Turns after Injection: 100
- v_x / v_y (nominal tunes): 6.7/6.8
- Q_s : 0.04295
- Space Charge: 2.5 D
- No. of Longitudinal Bins: 32
- No. of Transverse Bins: 64
- Beam Pipe Radius, \emptyset : 2.54e-2 (m)

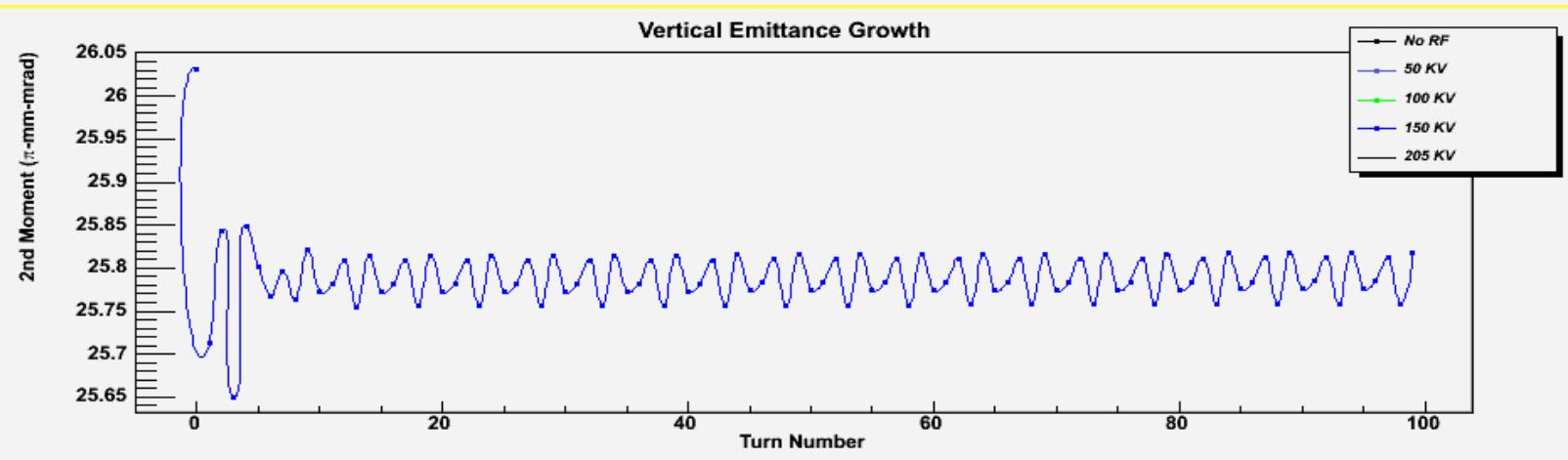
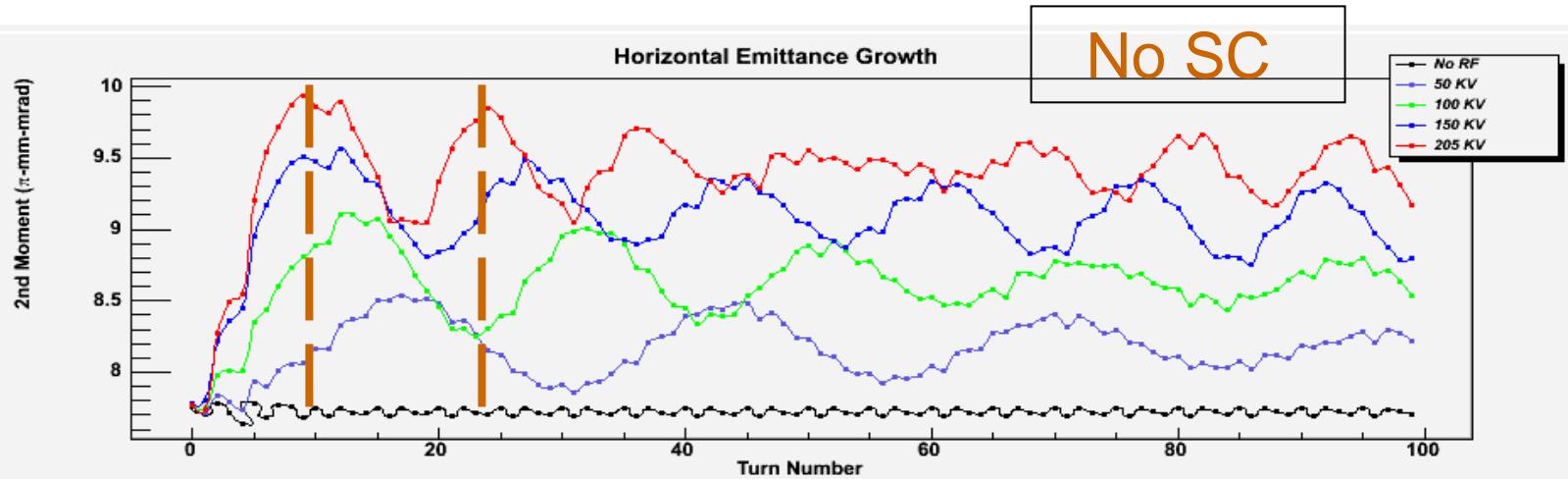
Change in the 2nd Moment Calculation

- Total x was replaced with x_β .
- β -function Normalization was removed.

$$x_\beta = x - D \cdot \delta \quad \left(\delta \equiv \frac{\Delta P}{P} \right)$$

$$\frac{\langle (x - \langle x \rangle)^2 \rangle}{\sqrt{\beta^2}} \Rightarrow \langle (x_\beta - \langle x_\beta \rangle)^2 \rangle$$

Emittance Growth with different RF Voltages



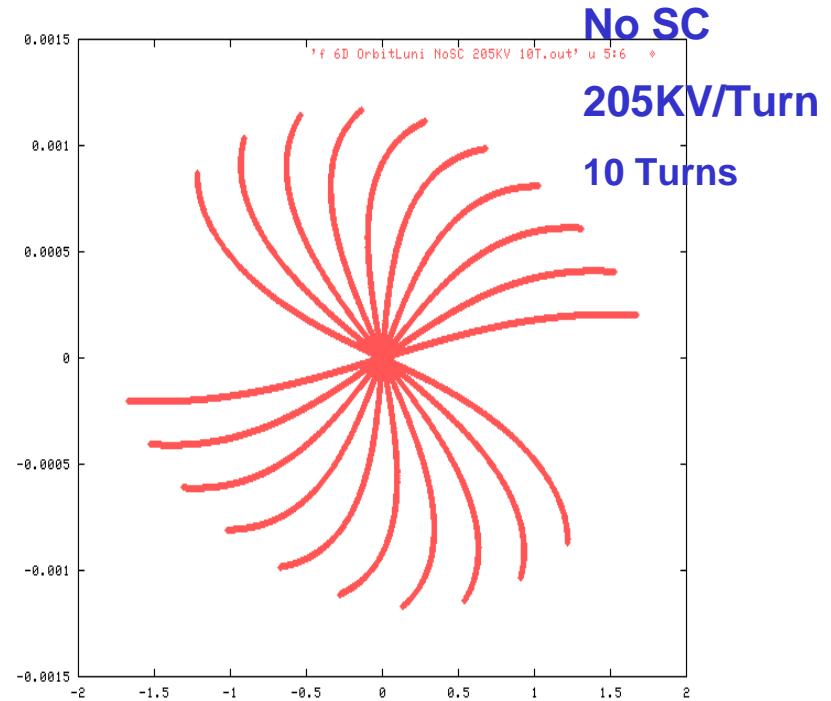
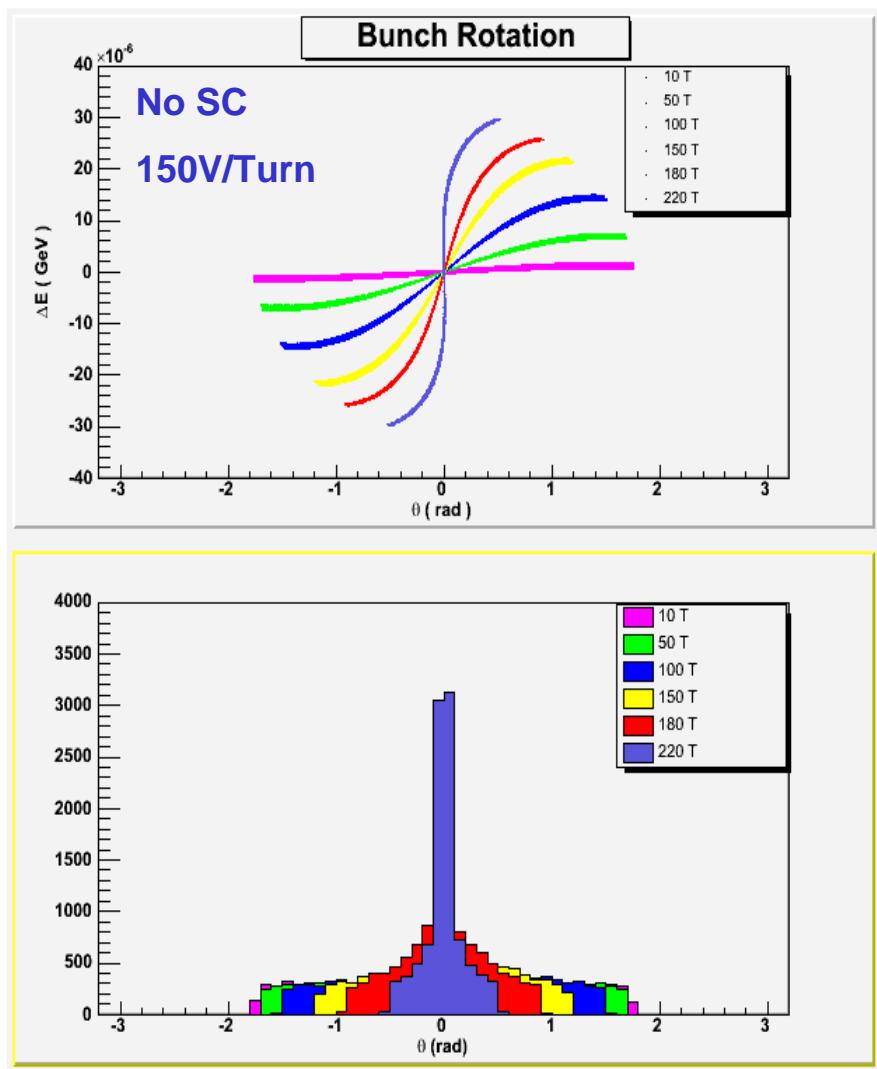
- 6x6 Transport Map

$$\begin{pmatrix} \begin{pmatrix} \mathbf{x}_f \\ P_x / P_s \end{pmatrix}_f \\ \begin{pmatrix} \mathbf{y}_f \\ P_y / P_s \end{pmatrix}_f \\ (-c \cdot \Delta t)_f \\ \begin{pmatrix} \Delta E \\ P_s C \end{pmatrix}_f \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \mathbf{x}_i \\ P_x / P_s \end{pmatrix}_i \\ \begin{pmatrix} \mathbf{y}_i \\ P_y / P_s \end{pmatrix}_i \\ (-c \cdot \Delta t)_i \\ \begin{pmatrix} \Delta E \\ P_s C \end{pmatrix}_i \end{pmatrix}$$

$P_s = P_0 \cdot (1 + \delta)$

$$\left\{ \begin{array}{l} \mathbf{x}_\beta = \mathbf{x} - D \cdot \delta \\ \mathbf{y}_\beta = \mathbf{y} \\ \\ \mathbf{x}_f = \cos \mathbf{k}_x \cdot L) \cdot \mathbf{x}_i + \left\{ \frac{\sin \mathbf{k}_x \cdot L)}{\mathbf{k}_x} \right\} \cdot \left(\frac{\mathbf{P}_x}{P_s} \right)_i + \left(\frac{1}{\rho \cdot \mathbf{k}_x^2} \right) \cdot (1 - \cos \mathbf{k}_x \cdot L) \cdot (\beta \cdot \delta) \\ \mathbf{y}_f = \cos \mathbf{k}_x \cdot L) \cdot \mathbf{y}_i + \left\{ \frac{\sin \mathbf{k}_y \cdot L)}{\mathbf{k}_y} \right\} \cdot \left(\frac{\mathbf{P}_y}{P_s} \right)_i \end{array} \right.$$

Micro-Bunch Rotation



- Bunch rotation produces a large momentum spread, and short bunches with large space-charge forces.
- with a higher RF Voltage, bunch rotation speeds up.

Observation on RF Effect

$$\frac{T_s}{T_0} = \frac{\omega_0}{\omega_s} = \sqrt{\frac{2 \cdot \pi \cdot \beta^2 \cdot E}{h \cdot \eta \cdot eV_{RF}}}$$

V _{RF}	(1/2)*(T _s /T ₀)
50 KV	24
100 KV	17
150 KV	14
205 KV	12

- Oscillation periods with various RF voltages in transverse emittance growth is about a half of the synchrotron period.